Results from a Conical Euler Methodology Developed for Unsteady Vortical Flows

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Abstract

N recent years, the understanding and prediction of the complex flows about high-performance aircraft at high angles of attack have generated much interest within the fluid dynamics community. These aircraft typically have thin, highly swept lifting surfaces that produce vortical flow over the leeward side of the vehicle at high angles of attack. This vortical flow can have beneficial effects on performance, such as lift augmentation at high angles of attack. However, it also may have adverse effects, such as structural fatigue due to tail buffet and stability and control problems due to wing rock, wing drop, nose slice, and pitch-up. Consequently, considerable experimental work has been done to understand the basic flow physics of vortical flows about delta wings at high angles of attack. Use of the more modern computational fluid dynamics techniques for the prediction of vortex-dominated flows has focused primarily on steady applications; there are notable exceptions where applications have been made to rolling delta wings undergoing forced harmonic motions.¹⁻³ The objective of the current research is to study unsteady, vortex-dominated flowfields by using the conical Euler equations as a first step in investigating the three-dimensional problem. The purpose of this paper is to report on the development of a conical Euler analysis method to study unsteady, vortex-dominated flows about rolling delta wings undergoing either pulsed motion, forced harmonic motion, or free-to-roll motion. The full paper gives descriptions of the conical Euler flow solver and free-to-roll analysis. The flow solver involves a multistage, Runge-Kutta time-stepping scheme and a cellcentered, finite volume spatial discretization of the Euler equations on an unstructured grid of triangles. The code was modified to include the simultaneous time integration of the rigid-body equation of motion with the governing flow equations to allow for the additional analysis of the free-to-roll case. Results that demonstrate how the systematic analysis of the forced response of the delta wing can be used to predict the stable, unstable, and neutrally stable free response of the delta wing are presented in this Synoptic. The conical Euler methodology developed is directly extendable to three-dimensional calculations.

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Calculations were performed for a 75-deg swept delta wing at a freestream Mach number of 1.2 and at $\alpha = 10$ and 30 deg. The grid, which was generated using an advancing-front method, has a total of 4226 nodes and 8299 elements. The grid was designed to be fine on the leeward side of the wing where

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the dominant flow features are expected to occur. For unsteady applications, the mesh is rotated as a rigid body to conform to the instantaneous position of the wing. Steady-state solutions are first calculated to provide starting solutions for the unsteady cases. Next, a pulse transfer-function analysis was performed to determine the linear stability characteristics of the delta wing roll response at each angle of attack. Using the pulse analysis, the rolling moment coefficient due to roll was determined over a wide range of reduced frequencies. The results of this analysis indicated that at $\alpha=10$ deg the free response of the delta wing would tend to be stable for small roll angles. However, at $\alpha=30$ deg, the free response of the delta wing would tend to be unstable for small roll angles.

Because the pulse analysis is limited to small perturbations (due to the linearity assumptions), the large perturbation aerodynamic response characteristics of the rolling delta wing are determined using a forced harmonic analysis. The forced harmonic rolling motion can be expressed as

$$\phi(\bar{t}) = \phi_0 \sin(k\bar{t}) \tag{1}$$

where ϕ_0 is the roll amplitude, k is the reduced frequency of oscillation (based on one-half of the wing root chord), and \bar{t} is the nondimensional time. Since the linear techniques are no longer applicable, the concept of energy transferred to the system can be used in this analysis to determine the stability characteristics. During one cycle of harmonic motion, the total aerodynamic energy added to the system is

$$\Delta E = \oint C_{\ell} \, \mathrm{d}\phi \tag{2}$$

where ΔE is a nondimensional energy, and C_ℓ is the rolling moment coefficient. Equation (2) indicates that, for the rolling moment coefficient vs roll angle response that traces a clockwise loop during one cycle of motion, the energy exchange is positive during the cycle. Similarly, for a counter-

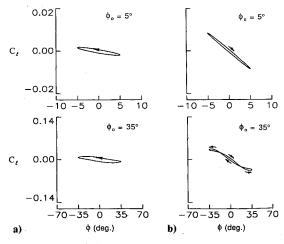


Fig. 1 Angle-of-attack effects on the rolling moment coefficient vs instantaneous roll angle for a 75-deg swept delta wing at $M_{\infty}=1.2$ and k=0.25: a) $\alpha=10$ deg, and b) $\alpha=30$ deg.

clockwise loop, the energy exchange is negative. If multiple loops are formed, then ΔE is a total of the individual loops.

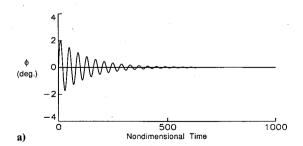
Motions at a reduced frequency of k = 0.25 were chosen for this analysis. This value lies at the midpoint of the range of reduced frequency identified by the pulse analysis as being an unstable condition for the free-to-roll wing at $\alpha = 30$ deg. Two amplitudes of motion, $\phi_0 = 5$ and 35 deg, were considered at $\alpha = 10$ and 30 deg. A comparison of rolling moment coefficient vs roll angle for each of these cases is shown in Fig. 1 to illustrate the effects of roll amplitude and angle of attack. For the $\alpha = 10$ deg cases shown in Fig. 1a, the results indicate a counterclockwise loop for each roll amplitude that would produce a convergent (stable) response if the wing were free to roll. This prediction of a stable response at the smallest roll amplitude is consistent with the pulse transfer-function results. As the roll amplitude is further increased to $\phi_0 = 35 \text{ deg}$, some nonlinear aerodynamic characteristics are exhibited in the "pinching" of the loop at the extreme roll angles, although the free-to-roll response still is predicted to be stable. For the $\alpha = 30$ deg cases shown in Fig. 1b, the results indicate a clockwise loop for the $\phi_0 = 5$ deg roll amplitude that would produce a divergent (unstable) response if the wing were free to roll. This prediction of an unstable, free-to-roll response at the smaller roll amplitude is also consistent with the pulse transfer-function results. For $\phi_0 = 35$ deg, counterclockwise loops have formed at the extreme roll angles that, consequently, would have a stabilizing effect on the free-to-roll response. The formation of these stabilizing loops was not predicted by the pulse analysis. In contrast with the $\alpha = 10 \deg$ case, at $\alpha = 30$ deg the nonlinear aerodynamic effects at the larger roll amplitudes result in a change in the stability characteristics of the wing.

The nondimensional equation of motion for a rolling delta wing can be expressed as

$$\phi'' = C_1 C_\ell - C_2 \phi' \tag{3}$$

$$C_1 = \frac{M_{\infty}^2 S c^3 \rho_{\infty}}{2I_{xx}} \qquad C_2 = \frac{\mu_x c}{a_{\infty} I_{xx}}$$
 (4)

where ϕ is the roll angle that is positive clockwise when viewed from aft, I_{xx} is the mass moment of inertia about the longitudinal axis, ℓ is the aerodynamic rolling moment, also positive clockwise, and μ_x is a structural damping term. Prime superscripts indicate differentiation with respect to nondimensional time \bar{t} . Time is nondimensionalized by the root chord of the



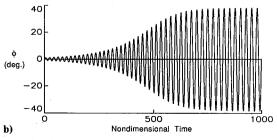


Fig. 2 Angle-of-attack effects on free-to-roll response for a 75-deg swept delta wing at $M_{\infty} = 1.2$: a) $\alpha = 10$ deg, and b) $\alpha = 30$ deg.

Table 1 Summary of structural parameter values and flow conditions for the free-to-roll calculation

Parameter	Value
c, m	0.2820
I_{xx} , kg m ²	0.1776×10^{-3}
μ_x , kg m ² /s	0.0
ρ_{∞} , kg/m ³	0.526
a_{∞} , m/s	312

delta wing c and the freestream speed of sound a_{∞} . The rolling moment coefficient is defined as

$$C_{\ell} = \frac{\ell}{q_{\infty} Sc} \tag{5}$$

where q_{∞} is the freestream dynamic pressure, and S is the planform area. The structural damping term is added to simulate the damping that might be provided by a sting balance bearing mount. The solution procedure for the time integration of Eq. (5) is based on a second-order-accurate, finite difference approximation of the time derivatives. After substituting these expressions into Eq. (5), the roll angle at time level n+1 can be expressed in terms of the roll angle at previous time levels as

$$\phi^{n+1} = \frac{C_1 C_\ell^{n+1} \Delta \bar{t}^2 + (5 + 2C_2 \Delta \bar{t}) \phi^n - (4 + \frac{1}{2} C_2 \Delta \bar{t}) \phi^{n-1} + \phi^{n-2}}{\frac{3}{2} C_2 \Delta \bar{t} + 2}$$
(6)

The rolling moment C_ℓ^{n+1} at time level n+1 is estimated from a linear extrapolation of C_ℓ at the previous two time levels. This predicted value of C_ℓ is used to determine the roll angle ϕ^{n+1} at time level n+1. The flowfield about the wing at this roll angle then is calculated, and the actual value of the rolling moment coefficient is determined. The rolling moment coefficient then is updated for use in the next time step. Because of the explicit time marching of the Euler code used in this study, the time steps required for numerical stability were very small. Thus, it was not necessary to iterate between the roll angle calculation and the flowfield calculation at each time step. For a free-to-roll calculation, steady-state initial conditions are specified for ϕ^{-1} , ϕ^0 , C_ℓ^{-1} , and C_ℓ^0 . At $\bar t=0$, an angular velocity perturbation is applied to the wing.

The free-to-roll results were obtained for the flow conditions and structural and inertial parameter values listed in Table 1. The initial angular velocity imposed on the wing was $\phi' = 0.003$, and the nondimensional time step was $\Delta t = 0.004$. The resulting roll angle response for the $\alpha = 10$ deg case, shown in Fig. 2a, indicates that after the initial perturbation the oscillatory response converges to its initial steady-state value. This stable free-to-roll response is consistent with the pulse and forced harmonic results. The roll angle response for the $\alpha = 30$ deg case, shown in Fig. 2b, indicates that initially the oscillatory response diverges for small values of roll angle, which is consistent with the small amplitude pulse and harmonic results. As the roll angle increases to approximately $\phi = 35$ deg, the rate of divergence decreases due to the stabilizing aerodynamics (counterclockwise loops in the rolling moment coefficient at the extreme roll angles) shown in Fig. 1b. Finally, the response reaches a maximum amplitude of motion at approximately $\phi = 38$ deg corresponding to a limit cycle oscillation. The reduced frequency of the limit cycle is k = 0.103.

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